Some applications of differentiation

Rates of change

If a quantity y depends on and varies with a quantity x then the rate of change of y with respect to x is $\frac{dy}{dx}$.

Thus, for example, the rate of change of pressure p with height h is $\frac{dp}{dh}$.

A rate of change with respect to time is usually just called 'the rate of change', the 'with respect to time' being assumed. Thus, for example, a rate of change of current, i, is $\frac{di}{dt}$ and a rate of change of temperature, θ, is $\frac{d\theta}{dt}$, and so on.

Problem 1. The length l metres of a certain metal rod at temperature θ°C is given by: $l = 1 + 0.00005\theta + 0.0000004\theta^2$. Determine the rate of change of length, in mm/°C, when the temperature is (a) 100°C and (b) 400°C

Solution

The rate of change of length means $\frac{dl}{d\theta}$

Since length $l = 1 + 0.00005\theta + 0.0000004\theta^2$,

then $\frac{dl}{d\theta} = 0.00005 + 0.0000008\theta$

(a) When $\theta = 100$°C,

$$\frac{dl}{d\theta} = 0.00005 + (0.0000008)(100) = 0.00013 \text{ m/°C} = \mathbf{0.13\ mm/°C}$$

(b) When $\theta = 400$°C,

$$\frac{dl}{d\theta} = 0.00005 + (0.0000008)(400) = 0.00037 \text{ m/°C} = \mathbf{0.37\ mm/°C}$$

Problem 2. The luminous intensity I candelas of a lamp at varying voltage V is given by: $I = 4 \times 10^{-4}\ V^2$. Determine the voltage at which the light is increasing at a rate of 0.6 candelas per volt

Solution

The rate of change of light with respect to voltage is given by $\frac{dI}{dV}$

Since $I = 4 \times 10^{-4}\ V^2$, $\frac{dI}{dV} = (4 \times 10^{-4})(2)V = 8 \times 10^{-4}\ V$

When the light is increasing at 0.6 candelas per volt then $+0.6 = 8 \times 10^{-4}\ V$, from which, voltage

$$V = \frac{0.6}{8 \times 10^{-4}} = 0.075 \times 10^{+4} = \mathbf{750\ volts}$$

Problem 3. Newtons law of cooling is given by: $\theta = \theta_0 e^{-kt}$, where the excess of temperature at zero time is θ_0°C and at time t seconds is θ°C. Determine the rate of change of temperature after 40 s, given that $\theta_0 = 16$°C and $k = -0.03$

Solution

The rate of change of temperature is $\frac{d\theta}{dt}$

Since $\theta = \theta_0 e^{-kt}$ then $\frac{d\theta}{dt} = (\theta_0)(-k)e^{-kt}$

$= -k\theta_0 e^{-kt}$

When $\theta_0 = 16$, $k = -0.03$ and $t = 40$ then

$$\frac{d\theta}{dt} = -(-0.03)(16)e^{-(-0.03)(40)}$$

$$= 0.48\, e^{1.2} = \mathbf{1.594°C/s}$$

Class work

1. An alternating current, i amperes, is given by $i = 10\sin 2\pi ft$, where f is the frequency in hertz and t the time in seconds. Determine the rate of change of current when $t = 20$ ms, given that $f = 150$ Hz. [3000π A/s]

2. The luminous intensity, I candelas, of a lamp is given by $I = 6 \times 10^{-4}\, V^2$, where V is the voltage. Find (a) the rate of change of luminous intensity with voltage when $V = 200$ volts, and (b) the voltage at which the light is increasing at a rate of 0.3 candelas per volt. [(a) 0.24 cd/V (b) 250 V]

ASIGNMENT/ HOME TASK

3. The voltage across the plates of a capacitor at any time t seconds is given by $v = V\, e^{-t/CR}$, where V, C and R are constants. Given $V = 300$ volts, $C = 0.12 \times 10^{-6}$ farads and $R = 4 \times 10^6$ ohms find (a) the initial rate of change of voltage, and (b) the rate of change of voltage after 0.5 s. [(a) -625 V/s (b) -220.5 V/s]